

An evaluation is made of the two new and three previously known formulas for the directional absorption capacity of an isothermal volume of water vapor at temperature T for a flux with a black spectrum at temperature T_* — $a_0(T_*, T, x)$.

The present study will compare and evaluate five different engineering formulas for the directional absorption capacity of water vapor, two of which are presented for the first time. Such formulas are not only important because they are used to find the absorption of radiation from black and gray surfaces by a gas. The absorption capacity or "blackness" of the gas is related in a special manner to the quantity a_0 . The function a_0 is used in calculating the radiation of a nonisothermal volume. The intensity of the radiation from the volume

$$I = \frac{\sigma}{\pi} \int_0^{x_0} T^4(x) \left. \frac{\partial a_0 [T(y), \bar{T}, x]}{\partial x} \right|_{y=x} dx,$$

where T is the local temperature at a depth x , equivalent to the temperature T_* in the general formula $a_0(T_*, T, x)$. The temperature \bar{T} is equivalent to T in the general formula for a_0 and is taken as a mean-weighted value over the segment $[0; x]$ in the first approximation. The approach of $\bar{T} \rightarrow T$ implies $a_0 \rightarrow \varepsilon$ and corresponds to the well-known Nevskii-Hottel-Ivantsov approximation.

Finally, an approach is known [1] through which the function $a_0(T_*, T, x)$ makes it possible to find the absorption capacity of the gas for a nonblack incident flux.

The world literature has almost exclusively recognized Hottel's formula, based on experimental data [2]:

$$a_0 = (T/T_*)^{0.45} \varepsilon [T_*; x(T/T_*)^{-1}]. \quad (1)$$

Using the same experimental data, V. N. Timofeev obtained the formula

$$a_0 = (T/T_*)^{0.4} \varepsilon (T, x). \quad (2)$$

In contrast to the first formula, the second has an untransformed argument — the emissivity — thus simplifying calculation of heat transfer of a gray body with the medium as much as is possible.

It is significant that the experimental base of Hottel's formula has been accepted as the most reliable up until the present, although other similar experimental studies did appear in the literature later. Therefore, in the present study, Eq. (1) will serve as a reference in the experimental data region where it was based. At the same time, there is no doubt that it gives inaccurate results at the limits of low and high optical thickness.

In [1], following Penner [3], an attempt was made to justify Hottel's formula theoretically. The conditions under which the absorption capacity has the form

$$a_0 = (T/T_*)^m \varepsilon [T_*; x(T/T_*)^u], \quad u = \chi - m - 1 \quad (3)$$

were clarified.

The meanings of all exponents were clarified, and the factor χ was introduced for the first time; m and χ were averaged over the bands of the spectrum; m defines the degree of the temperature dependence of the bandwidth parameter with averaging over the spectral bands. Water vapor is characterized by a quite strong overlapping of its bands, especially with increase in temperature. The bands do not show free broadening. Therefore, with growth in temperature, m decreases, and this fact was clearly demonstrated even in the

experiments of Hottel and Egbert [2]. Ludwig [4] has presented the most complete picture of band overlap with growth in temperature from 300 to 3000°K. Using this material, the present authors obtained the estimate

$$m = 0.1 \exp(1.83 - 0.327\theta), \quad \theta = T/1000. \quad (4)$$

The factor χ expresses the degree of the temperature dependence of integral radiation intensity of a single molecule. For an optically thin medium, the radiation of which is determined almost completely by fundamental bands, $\chi = 0$. With increase in optical thickness, an ever greater contribution to radiation at the boundaries of the volume is made by weak upper state bands for which $\chi > 0$. We estimate that

$$\chi = 0.5 [1 - \exp(-5x)]. \quad (5)$$

It should be stressed that the factor χ is definitely temperature dependent, and its dependence on optical thickness given here is coarse. Nevertheless, Eq. (5) does give a correct correction to Eq. (1).

The fourth formula, like Eq. (3), is derived from the general formula for absorption capability of a medium with a vibrational-rotational spectrum,

$$a_0 = \frac{\pi}{\sigma T_*^4} \sum_j I_{0j}(T_*) A_j(T). \quad (6)$$

The simplification consists of averaging integral absorption capacity A_j over spectral bands and bringing it ahead of the summation symbol,

$$a_0 = A(T) \frac{\pi}{\sigma T_*^4} \sum_j I_{0j}(T_*).$$

For limitingly small optical thicknesses,

$$\pi \sum_j I_{0j}(T_*) / \sigma T_*^4 = \alpha(T_*) \bar{S}(T_*),$$

where α is the mean Planck absorption coefficient; \bar{S} is the integral intensity of a band averaged over bands. In the temperature dependence

$$\bar{S} = \bar{S}_0 (T_*/T_0)^{\chi-1}$$

the exponent χ proves to be averaged over the spectrum in some manner. Equation (6) takes on the form

$$a_0 = \frac{\alpha(T_*)}{\bar{S}_0 (T_*/T_0)^{\chi-1}} A(T, x). \quad (7)$$

As $T_* \rightarrow T$

$$a_0 \rightarrow \varepsilon(T) = \frac{\alpha(T)}{\bar{S}_0 (T/T_0)^{\chi-1}} A(T, x). \quad (8)$$

Comparison of the last formulas gives the final expression

$$a_0 = \frac{\alpha(T_*)}{\alpha(T)} \left(\frac{T}{T_*} \right)^{\chi-1} \varepsilon(T, x). \quad (9)$$

The argument of the formula coincides with that used by Timofeev. The factor before the emissivity differs. First of all, it was chosen for limitingly low optical thickness. Secondly, the approach of averaging absorption over spectral bands is, of course, coarse. Therefore, the exponent χ , still subject to definition, must not be obtained from relationships for individual bands, but from other data. In the limit of small optical thickness $\chi \rightarrow 0$ and Eq. (9) gives a correct result. For the limit of high optical thickness,

$$a_0 \rightarrow \varepsilon \rightarrow 1 \text{ and } [\alpha(T_*)/\alpha(T)] (T/T_*)^{\chi-1} \rightarrow 1.$$

We have determined the mean Planck absorption coefficient from new experimental data [5], which is approximated well by the formula

$$\lg \alpha = 1.7376 - 1.1374\theta + 0.158\theta^2, \quad (10)$$

where $\theta = T/1000$. With this value we obtain $\chi_\infty \approx -1.1$. The limiting values thus obtained were interpolated with the formula

$$\chi = -0.156 [1 - \exp(-6.2x)], \quad (11)$$

which is a very coarse estimate. There is no doubt that the exponent χ also depends on temperatures T_* and T . However, refinement of its value is not yet possible.

Equation (9) with Eqs. (10) and (11) will now be compared with all the other formulas for absorption capacity of water vapor.

All the formulas considered above are based directly or indirectly on the experimental material of Hottel and Egbert. The fifth and last formula, that of Leckner [6], is based on a new mass of spectroscopic data published in American sources:

$$a_0 = (T/T_*)^{1.25} \varepsilon [T_*; x(T/T_*)^{-2}]. \quad (12)$$

Leckner compared the a_0 values produced by Eq. (12) with the correct values obtained by summation of the spectral absorption capabilities of the gas, calculated from the same experimental data. In some cases the divergence proved to be significant. The exponents of Eq. (12) differ markedly from those used in the preceding group of formulas. In our opinion, Eq. (12) lacks justification.

It is desirable to compare all the formulas with a single calculation of their argument — the emissivity. We have used the emissivity formula published in [7]. Its advantage over other formulas which have been published is its simplicity and the convenience of extrapolation to any optical thickness, pressure, or temperature above 600°K. We will present the formula in its complete form, because of a misprint in the reference cited,

$$\varepsilon = \frac{\alpha \gamma_{\text{eff}}}{g \theta^u} \left[1 - \exp \left(- \frac{g \theta^u x}{\gamma_{\text{eff}} \sqrt{1 + z_* \gamma_{\text{eff}} g \theta^u x d} \sqrt{T/273/P_*}} \right) \right], \quad (13)$$

$$d = \exp(1.21 - 1.06\theta), \quad \theta = T/1000, \quad \gamma_{\text{eff}} = 1.5 \sqrt{\ln(4.9 + 1.1g\theta^u x)}.$$

where $\log \alpha = 0.9856 - 0.6366\theta_* - 0.0575\theta_*^2 + 0.0383\theta_*^3$, $\theta_* = t/1000$, $t = T - 273$, $z_* = 0.3$, $u = -1.25$, $g = 15.9$.

Control can be accomplished by using the value $\varepsilon = 0.197$ at $x = 1 \text{ m} \cdot \text{atm}$ and $T = 2000^\circ\text{K}$. One of the factors is somewhat simplified as compared to the form published in [7]. The formula as presented describes Hottel's experimental data for $T > 600 \text{ K}$ with sufficient accuracy.

We will evaluate the formulas at the limits of optical thickness. For limitingly low optical thicknesses it can be assumed that absorption is defined by the fundamental bands of the spectrum, those most intense, and then [3]

$$a_0 = x(T_*/T) \alpha(T_*).$$

The new formulas (3) and (9) satisfy the limit. With Hottel's Equation (1)

$$a_0 = x(T_*/T)^{0.55} \alpha(T_*),$$

and with Leckner's Equation (12)

$$a_0 = x(T_*/T)^{0.75} \alpha(T_*).$$

The true exponent is evidently somewhat less than unity, but much more than the values of 0.55 and 0.75 given above.

In the limit of high optical thickness, for the inequality $T > T_*$, Eqs. (1) and (12) can produce absorption capacities greater than unity. Equations (3) and (9) are practically free of this defect.

The five absorption capacity formulas presented were compared with numerical results obtained over wide argument ranges. Some of the results are presented in Table 1. Figure 1 shows curves given by the two more accurate formulas — Eqs. (1) and (3).

TABLE 1. Comparison of Five Formulas for Absorption Capacity of Water Vapor

T	x								
	0,5			1,0			3,0		
1000	T_*								
	1500	2000	3000	1500	2000	3000	1500	2000	3000
	202	144	110	252	179	134	337	234	169
	202	144	110	250	177	133	335	233	167
	195	138	099	245	174	125	332	236	170
	244	218	185	306	273	232	415	370	314
	167	103	061	204	123	070	266	154	082
1500	T_*								
	1000	2000	3000	1000	2000	3000	1000	2000	3000
	298	149	113	379	190	144	527	257	188
	293	151	116	375	192	148	519	260	194
	309	149	106	392	190	136	538	260	187
	246	187	159	313	238	202	430	326	277
	352	132	083	458	165	099	650	218	122
2000	T_*								
	1000	1500	3000	1000	1500	3000	1000	1500	3000
	304	213	114	392	276	150	554	385	202
	289	208	117	374	271	154	525	377	209
	314	213	108	403	277	141	563	383	195
	199	170	129	260	221	167	369	305	231
	389	236	098	528	313	123	782	448	158

Note. Each block of the table shows values given by (from top to bottom) Eq.(1), Eq. (3) [with consideration of Eqs.(4) and (5)], Eq.(9), Eq.(2) and Eq.(12). Emissivity was calculated with Eq.(13). Results in the second row are considered the best. Digits to the right of the decimal point are shown.

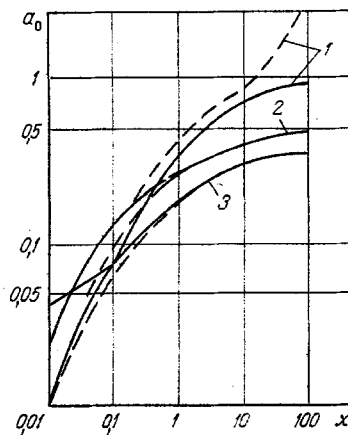


Fig. 1. Directional absorption capacity of water vapor at temperature T for flux with black spectrum at temperature T_* versus optical thickness x: dashed curves) Hottel's Equation (1); solid curves) Eq. (3) with consideration of Eqs. (4) and (5). 1) $T_* = 1000^\circ\text{K}$, $T = 4000^\circ\text{K}$; 2) $T_* = 4000^\circ\text{K}$, $T = 1000^\circ\text{K}$; 3) $T_* = 2000^\circ\text{K}$, $T = 1000^\circ\text{K}$. Significant divergence occurs in regions of low and high optical thickness.

Analysis of all results revealed that Eq. (3) with exponents m and χ defined by Eqs. (4) and (5) is the best. It describes the experimental data of Hottel and Egbert well. The values of m published in [2] for the temperature interval 645–1390°K are reproduced by Eq. (4). In Eq. (1), Hottel used an average value $m = 0.45$. In contrast to Eqs. (1), (11), and (12), Eq. (3) gives a correct limit for low optical thickness. In the high-optical-thickness limit the absorption capacity only slightly exceeds unity for the most unfavorable combination of arguments, while the values given by Eqs. (1) and (12) significantly exceed unity. This is shown in Fig. 1 at $T_* = 1000$ and $T = 4000^\circ\text{K}$.

NOTATION

a_0 is the directional absorption capacity of gas for flow with black spectrum, dimensionless;
 g is the correlation constant, $(\text{m} \cdot \text{atm})^{-1}$;
 x is the optical thickness for partial gas pressure, $\text{m} \cdot \text{atm}$;
 t is the temperature, $^\circ\text{C}$;
 A is the integral absorption in spectral band, cm^{-1} ;
 \bar{S} is the integral band intensity, $\text{cm}^{-1}/\text{m} \cdot \text{atm}$;
 I is the flux intensity at volume boundary, $\text{W}/\text{m}^2\text{sr}$;
 T, T_* are the temperatures of gas and radiating black body, $^\circ\text{K}$;
 α is the averaged Planck absorption coefficient, $(\text{m} \cdot \text{atm})^{-1}$;
 ε is the gas emissivity, dimensionless;
 γ is the bandwidth parameter, cm^{-1} ;
 I_{0j} is the Planck intensity, $\text{cm} \cdot \text{W}/\text{m}^2\text{sr}$;
 $\theta = T/1000$,

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